Exercise 8

If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.

(a) What is the maximum height reached by the ball?

(b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

Solution

Part (a)

To determine the velocity, take the derivative of the position function.

$$v(t) = \frac{ds}{dt}$$

$$= \frac{d}{dt}(80t - 16t^2)$$

$$= 80 - 32t$$

The ball reaches its maximum height when it comes to a standstill in the air, so set $v(t) = 0$ and solve the equation for t.

$$v(t) = 0$$

$$80 - 32t = 0$$

$$t = \frac{80}{32}$$

$$t = 2.5 \text{ s}$$

Part (b)

Start by finding out when the ball is 96 feet above the ground: Set $s(t) = 96$ and solve the equation for t.

$$s(t) = 96$$

$$80t - 16t^2 = 96$$

$$16t^2 - 80t + 96 = 0$$

$$16(t^2 - 5t + 6) = 0$$

$$16(t - 2)(t - 3) = 0$$

$$t = \{2, 3\}$$

Since the ball is thrown vertically upward, the ball is on its way up at $t = 2$ and is on its way down at $t = 3$. Plug these two times into the velocity function.

$$\text{On its way up:} \qquad v(2) = 80 - 32(2) = 16 \ \frac{\text{ft}}{\text{s}}$$

$$\text{On its way down:} \qquad v(3) = 80 - 32(3) = -16 \ \frac{\text{ft}}{\text{s}}$$